Task

Assume that the amount of money deposited in a bank is proportional to the square of the interest rate the bank pays on this money. Furthermore, the bank can reinvest this money at 12%.

Find the interest rate the bank should pay to maximize profit.

Solution

Amount of money is M.
Interest rate is i and $ki^2 = M$, where k is a constant.
The bank can invest at 12%.
Profit = $P(i) = 0.12M - iM$, but we know M is related to i by $M = ki^2$.
Putting in $M = ki^2$, we get $P(i) = 0.12ki^2 - ki^3$.
Taking the derivative gives us $DP/di = 0.24ki - 3ki^2 = ki(0.24 - 3i)$.
From here, we set the derivative to 0 to find the maximum, so we have $0.24 - 3i = 0$, which means i should be 0.08.

Answer: 8%.