Answer on Question #42157, Economics, Economics of Enterprise

A. If L = 1 and K = 1, the cost of producing any given output is minimized and Q = 1*1^2 = 1
Since the lower bound is valid for every $y \geq 0$, we can search for the best one, that is, the largest lower bound:
$$p^* \geq d^* := \max_{y \geq 0} \ g(y).$$

The problem of finding the best lower bound:

$$d^* := \max_{y \geq 0} \ g(y)$$

is called the *dual problem* associated with the Lagrangian defined above. It optimal value d^* is the *dual optimal value*. As noted above, g is concave. This means that the dual problem, which involves the maximization of g with sign constraints on the variables, is a convex optimization problem.

Example: For the problem of minimum distance to a polyhedron above, the dual problem is

$$d^* = \max_{y \geq 0} \ g(y) = \max_{y \geq 0} \ -b^T y - \frac{1}{2}\|A^T y\|_2^2.$$