Question #42139, Economics, Economics of Enterprise

Task:

The heights of a group of 1000 people are measured, and found to be distributed normally with mean height of 178 cm and a standard deviation of 12 cm.
How many people do we expect to find with a height of 178 cm or less ?

Solution:

Let ξ - is a random variable individual growth.

According to the problem stated, that ξ to be distributed normally with mean height of 178 cm and a standard deviation of 12 cm. It's means that:

$$\xi \sim \mathcal{N}(178, 12^2) - normal\ distribution\ with\ parameters\ 178\ and\ 12.$$

$$\xi \sim f_\xi(x) = \frac{1}{\sqrt{2\pi * 12^2}} e^{-\frac{(x-178)^2}{2*12^2}} = \frac{1}{12\sqrt{2\pi}} e^{-\frac{(x-178)^2}{288}}, where\ is\ f_\xi - density\ distribution$$

$$Also: \xi \sim F_\xi(x) = \int_{-\infty}^{x} f_\xi(t)\, dt, where\ is\ F_\xi - distribution\ function$$

$$Let\ S = \{h_1, \dots, h_{1000}\} - a\ growing\ set\ of\ 1000\ people$$

$$Let\ A = \{h_{i_1}, \dots, h_{i_k}\} - a\ growing\ set\ of\ people, whose\ height\ is\ less\ than\ 178cm$$

$$\mathbb{P}(\xi \leq 178) = \frac{|A|}{|S|} - probability\ that\ the\ person\ is\ less\ than\ 178cm$$

$$|A| = |S|\mathbb{P}(\xi \leq 178)$$

$$\mathbb{P}(\xi \leq 178) = F_\xi(178) = \int_{-\infty}^{178} f_\xi(t)\, dt = \int_{-\infty}^{178} \frac{1}{12\sqrt{2\pi}} e^{-\frac{(t-178)^2}{288}}\, dt$$

$$= \left[Let\ \frac{t-178}{12} = a, => t = 12a + 178, and\ dt = 12da \right]$$

$$= \int_{-\infty}^{0} \frac{12}{12\sqrt{2\pi}} e^{-\frac{a^2}{2}}\, da = \left[Let\ g(a) = e^{-\frac{a^2}{2}}, obviously, that\ g(a) = g(-a) => \right]$$

$$= \int_{0}^{\infty} \frac{12}{12\sqrt{2\pi}} e^{-\frac{a^2}{2}}\, da = \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-\frac{a^2}{2}}\, da = \left[Let\ a = \sqrt{2c}, then\ da = \frac{1}{\sqrt{2c}} dc \right]$$

$$= \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-c} \frac{1}{\sqrt{2c}} dc = \frac{1}{2\sqrt{\pi}} \int_{0}^{\infty} e^{-c} c^{\frac{1}{2}-1} dc = \frac{1}{2\sqrt{\pi}} \Gamma\left(\frac{1}{2}\right)$$

$$= [Where\ is\ \Gamma(\ ) - Euler\ gamma\ function] = \frac{1}{2\sqrt{\pi}} \sqrt{\pi} = \frac{1}{2}$$

$$|A| = |S| * \frac{1}{2} = \frac{1000}{2} = 500$$

Answer: 500 people with height less then 178cm.