a profit maximizing firm faces the following constrained maximization problem:p(x,y)=80x-2x^2-xy-3y^2+100y
subject to: x+y=12
Determine profit maximizing output level of commodities x and y subject to the conditions that output equals 12
Detailed explanation: YES

Solution.

Use the Lagrangian method to maximize the following profit function:
π = 80X – 2X^2 – XY – 3Y^2 + 100Y
Subject to the following constraint:
X + Y = 12 (output capacity constraint)
Set the constraint function equal to zero and obtain
0 = 12 – X –Y
Form the Lagrangian function
L = 80X – 2X^2 – XY – 3Y^2 + 100Y + λ(12 - X - Y)

Find the partial derivatives and solve simultaneously
∂L/∂X = 80 – 4X –Y - λ = 0
∂L/ ∂Y = – X – 6Y + 100 - λ = 0
∂L/ ∂λ = 12 - X – Y = 0
Subtract the second equation from the first equation, we get
- 3X + 5Y -20 = 0

Solve the system of equations

- 3X + 5Y -20 = 0

12 - X – Y = 0
Solution: X = 5, Y = 7, and λ = 53

Find $\partial^2 L/\partial X^2 = -4$, $\partial^2 L/\partial Y^2 = -6$, $\partial^2 L/\partial X\,\partial Y = -1$. Matrix $\begin{pmatrix} \partial^2 L/\partial X^2 & \partial^2 L/\partial X\,\partial Y \\ \partial^2 L/\partial X\,\partial Y & \partial^2 L/\partial Y^2 \end{pmatrix} = \begin{pmatrix} -4 & -1 \\ -1 & -6 \end{pmatrix}$ is negative definite because $\Delta_1 = -4 < 0$,

$\Delta_2 = \begin{vmatrix} -4 & -1 \\ -1 & -6 \end{vmatrix} = (-4)*(-6) - (-1)*(-1) = 24 - 1 = 23 > 0$. So, indeed, we have found profit maximizing output level of commodities X and Y.

So, the profit will be π = 80*5 – 2*5^2 – 5*7 – 3*7^2 + 100*7 = 400 - 50 - 35 - 147 + 700 = $868