Task: 4. Given the following production and cost function

Given by Y=15L0.4 K0.5 (Production function)

C=W1L+W2K+100 (Cost Function)

Where L= labor, K= capital,

A) Compute the following

i. APPL (2 points)

ii. APPK (2 points)

iii. MPPL (2 points)

iv. MPPK (2 points)

v. Marginal rate of technical substitution of labour for capital /MRTSLK(2 points)

vi. Expansion path (2 points)

vii. Elasticity of substitution (2 points)

viii. Return to scale (2 points)

ix. Indicate the relationship between elasticity of substitution and scale. (2 points)

Solution:

i. Average product (AP) is the total product divided by the number of units of variable factor used to produce it.

$$APPL = \frac{Y}{L}$$

$$APPL = \frac{15L^{0.4}K^{0.5}}{L} = 15L^{-0.6}K^{0.5} = \frac{15K^{0.5}}{L^{0.6}}$$

ii. We can find APPK is the same way:

$$APPK = \frac{Y}{K}$$

$$APPK = \frac{15L^{0.4}K^{0.5}}{K} = \frac{15L^{0.4}}{K^{0.5}}$$

iii. Marginal product (MP) is the change in total product resulting from the use of one additional unit of the variable factor.

$$MPPL = \frac{\partial Y}{\partial L} = 6\frac{K^{0.5}}{L^{0.6}}$$

iv. We can find MPPK in the same way as MPPL:

$$MPPK = \frac{\partial Y}{\partial K} = 7.5\frac{L^{0.4}}{K^{0.5}}$$

v. The Marginal Rate of Technical Substitution (MRTS) is the amount by which the quantity of one input has to be reduced ($-\Delta K$) when one extra unit of another input is used ($\Delta L = 1$), so that output remains constant (Y=const).

$$MRTS_{LK} = \frac{MPPL}{MPPK}$$

$$MRTS_{LK} = \frac{6K^{0.5}}{L^{0.6}} : \frac{7.5L^{0.4}}{K^{0.5}} = 0.8\frac{K}{L}$$

vi. As we have Cost Function given, we can find the expansion path. But first of all we should find prices for the resources. W_1 is the price for the labor, W_2 is the reward for the capital. Expansion path is the next:

$$\frac{MPPL}{MPPK} = \frac{W_1}{W_2}$$

$$\frac{0.8K}{L} = \frac{W_1}{W_2}$$

$$K = L\frac{W_1}{0.8W_2}$$

vii. The elasticity of substitution is:

$$E = \frac{\partial \ln(\frac{L}{K})}{\partial \ln(MRTS_{LK})}$$