

Task#85364

Given that the spacing of the lines in the microwave spectrum of $^{27}\text{Al}^1\text{H}$ is

1. Calculate the moment of inertia and bond length of—constant at 12.60 cm^{-1} the molecule.

Solution: The energy expression of rotational level is given by $E = \frac{J(J+1)h^2}{8\pi^2 I}$;

Selection rule for rotational transition is $\Delta J = \pm 1$;

The energy gap between two successive rotational level (ΔE) is given by

$$\Delta E = \frac{[(J+1)(J+2) - J(J+1)]h^2}{8\pi^2 I}, \Delta E = \frac{[J \times J + 3J + 2 - J \times J]h^2}{8\pi^2 I}; \Delta E = \frac{2(J+1)h^2}{8\pi^2 I}; ; \frac{\Delta E}{hc} = \frac{2(J+1)h}{8\pi^2 I \times c};$$

$$\frac{1}{\lambda} = \frac{2(J+1)h}{8\pi^2 I \times c} \dots\dots(1);$$

For J: $1 \rightarrow 0$ rotational level transition $J=0$ in equation (1)

$$\text{Let, } B = \frac{h}{8\pi^2 I \times c}, \frac{1}{\lambda} = 2B,$$

So, energy gap between two successive rotation levels $= 2B = 12.60 \text{ cm}^{-1}$, planck's constant $= 6.627 \times 10^{-27} \text{ erg.s}$, I = moment of inertia of the molecule. Where, $I = \mu r^2$, r = bond length of $^{27}\text{Al}^1\text{H}$.

$$B = \frac{h}{8\pi^2 I \times c} = 6.30 \text{ cm}^{-1}; I = \frac{6.627 \times 10^{-27}}{6.30 \times 8\pi^2 \times 3 \times 10^{10}}; I = 4.4 \times 10^{-40} \text{ gm.cm}^2$$

$$\text{Reduce mass of } ^{27}\text{Al}^1\text{H}(\mu) = \frac{M(^{27}\text{Al}) \times M(^1\text{H})}{M(^{27}\text{Al}) + M(^1\text{H})} = \frac{\left(\frac{27}{N}\right) \times \left(\frac{1}{N}\right)}{\frac{27}{N} + \frac{1}{N}} = \frac{27}{6.023 \times 10^{23} \times 28} = 1.6 \times 10^{-24} \text{ gm};$$

Where, N = Avogadro's number, $M(^{27}\text{Al})$ = Mass of an ^{27}Al atom,

$$I = \mu r^2, r = (I/\mu)^{1/2} = \frac{4.4 \times 10^{-40}}{1.6 \times 10^{-24}} = 1.6576 \times 10^{-8} \text{ cm} = 1.6576 \text{ \AA}$$

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