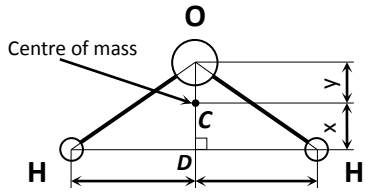
Answer on Question #41550 - Chemistry - Inorganic Chemistry

Question:
The distance between the oxygen molecule and each of the hydrogen atoms in a water (H_2O) molecule is 0.96 Å and the angle between the two oxygen-hydrogen bonds is 105°. Treating the atoms as particles, find the centre of mass of the system.

Solution:
A water molecule may be represented as follows



Thus, we have an isosceles triangle ΔHOH, where
HO = 0.96 Å
angle ∟HOH = 105°
Let us designate the centre of mass of the system in figure above as ***C***.
Since water molecule is symmetric the center of mass lies on the axis of symmetry, i.e. it is equidistant from the hydrogen atoms (H). Let us designate the midpoint between H atoms as ***D*** (HD = HH/2).
Thus, in given case to find the centre of mass of the system means to find values of HD, DC and CO.
In rectangular triangle ΔDOH angle ∟DOH = ∟HOH/2 = 105°/2 = 52.5°
OD = HO·cos(∟DOH) = 0.96·cos(52.5°) = 0.96 · 0.609 = 0.58 Å
HD = HO·sin(∟DOH) = 0.96·sin(52.5°) = 0.96 · 0.793 = 0.76 Å
DC + CO = OD = 0.58 Å
Molar mass of O atom M_O = 16.00 g/mol, molar mass of H atom M_H = is 1.01 g/mol.
In consideration of law of the lever:
$CO \cdot M_O = DC \cdot 2M_H$
Assigning DC = x, CO = y and substituting the known values we get the system of two equations:

$$\begin{cases} x + y = 0.58 \\ 16.00 \cdot y = 2 \cdot 1.01 \cdot x \end{cases}$$

$y = 2.02 \cdot x / 16.00 = 0.126 \cdot x$
$x + 0.126 \cdot x = 0.58$
$1.126 \cdot x = 0.58$
$x = 0.58/1.126 = 0.51$
$y = 0.58 – x = 0.58 – 0.51 = 0.07$

So, DC = 0.07 Å and CO = 0.51 Å.

Answer