

Answer on Question #38859, Chemistry, Other

Question

If 20 g of steam at 100°C were added to 50 g of water and 50 g of ice at 0°C, find the final temperature.

Solution

Water heat of vaporization $h_v = 2230 \text{ J/g}$, water heat of fusion $h_f = 334 \text{ J/g}$, heat capacity of water $C = 4.2 \text{ J/(g}\cdot\text{K)}$.

A system of ice and water will stay at 0 °C while receiving external heat until all ice is melted. This heat will be released from condensation of steam (h_v per each gram) and from further cooling of condensed water (C per each gram per each °C). And only if the heat that can be released from the steam will be larger than that needed to melt ice, the excess of heat will be redistributed in water, raising the final temperature above 0 °C.

Calculating how much heat is needed to melt all the ice:

$$Q_1 = m_{ice} h_f, Q_1 = 50 \text{ g} \cdot 334 \text{ J/g} = 16700 \text{ J}.$$

Calculating how much heat can release the steam being condensed and subsequently cooled to 0 °C:

$$Q_2 = m_{steam} h_v + m_{steam} C \Delta T, Q_2 = 20 \text{ g} \cdot 2230 \text{ J/g} + 20 \text{ g} \cdot 4.2 \text{ J/(g}\cdot\text{K)} \cdot (100 - 0) = 53000 \text{ J}.$$

$Q_2 > Q_1$, hence all ice will melt and the final temperature will be higher than 0 °C. The excess of heat after melting the ice will be $Q_2 - Q_1$ and will be redistributed in the total of 120 g (20 + 50 + 50) of water, this gives us a final temperature of water:

$$\Delta T = T_{final} - 0^\circ \text{C} = \frac{Q_2 - Q_1}{m_{total} C}, T_{final} = (53000 - 16700) / (120 \cdot 4.2) = 72^\circ \text{C}.$$

Answer: 72 °C.