Question

The first ionization energy of sodium is 496 kJ/mol. Use Coulomb's law to estimate the average distance between the sodium nucleus and the 3s electron. How does this distance compare to the atomic radius of sodium? Explain the difference.

Solution

First ionization energy of sodium is equal to the energy of attraction of 3s electron and the nucleus.

$$E = F \cdot r,$$

Where F – electrostatic force, r – distance between the sodium nucleus and the electron

$$r = \frac{E}{F}$$

Convert the ionization energy from [kJ/mol] to [J](per atom)

$$E = \frac{E^*}{N_A} = \frac{4.96 \cdot 10^5}{6.022 \cdot 10^{23}} = 8.24 \cdot 10^{-19}\, J$$

According to the Coulomb's law

$$F = k\frac{|q_1| \cdot |q_2|}{r^2},$$

where k - Coulomb's constant (k = $9 \cdot 10^9$ $N \cdot m^2 \cdot C^{-2}$), q_1 – electron charge ($q_1 = q_e$ = $-1.6 \cdot 10^{-19}$C), q_2 – sodium nucleus charge. q_2 = $11 \cdot (-q_e) = 1.76 \cdot 10^{-18}$C.

Thus, we may write

$$r = \frac{E}{F} = \frac{E \cdot r^2}{k \cdot |q_1| \cdot |q_2|},$$

whence

$$r = k\frac{|q_1| \cdot |q_2|}{E} = \frac{9 \cdot 10^9 \cdot 1.6 \cdot 10^{-19} \cdot 1.76 \cdot 10^{-18}}{8.24 \cdot 10^{-19}} = 3.08 \cdot 10^{-9} m$$

The atomic radius of sodium $r_{Na} = 0.186 \cdot 10^{-9} m$, that is almost 17 times less than the result obtained.

Such big difference is probably because the Coulomb's law considers stationary objects, while electrons in the atom move very fast. The influence of other electrons is not taken into account too. The Coulomb's law can be directly applied to the relationship between the nucleus and the first electron. But it alone cannot describe the amount of force between the nucleus and subsequent outer electrons as, to some degree, they are shielded from the force by the inner electrons. Besides, atomic radius cannot be calculated precisely without regard to quantum theory and in particular to electron shell theory.

Answer $r = 3.08 \cdot 10^{-9} m$