Sample: Trigonometry - Properties of Trigonometric Functions

1 Math Homework.

Complete your assignment and submit it to your instructor. Fill in the following table for \( f(x) = \cos(x) \) (each blank in the table is worth 1 point):

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2( \pi )</th>
<th>-3( \pi/2 )</th>
<th>-( \pi )</th>
<th>-( \pi/2 )</th>
<th>0</th>
<th>( \pi/2 )</th>
<th>( \pi )</th>
<th>3( \pi/2 )</th>
<th>2 ( \pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \cos(x) )</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Graph the two periods of the cosine function from the table (graph is worth 9 points):

[Graph of cosine function]

Fill in the following properties of the graph of the cosine function. To receive full credit for each question, you must explain your answer (each problem is worth 2 points):

1. Domain of \( f(x) = \cos(x) \).
   Domain of \( f(x)=\cos(x) \) is \( (-\infty, \infty) \), because argument \( x \) can take on any real value.

2. Range of \( f(x) = \cos(x) \).
   Range \( f(x)=\cos(x) \) is \([-1,1]\), because \( \cos(x) \) always less or equal then 1 and greater or equal then -1.

3. Period of \( f(x) = \cos(x) \).
   Period of \( f(x)=\cos(x) \) is \( 2\pi \), because \( 2\pi \) is the smallest value \( p \) for which \( \cos(x + p) = \cos(x) \) for all \( x \).

4. The x-intercepts of \( f(x) = \cos(x) \).
   The x-intercepts of \( f(x) = \cos(x) \) are \( x = \frac{\pi}{2} + \pi n \) where \( n \) is an integer (positive or negative), because \( \cos(\frac{\pi}{2} + \pi n) = 0 \).

5. The y-intercept of \( f(x) = \cos(x) \).
The y-intercept of \( f(x) = \cos(x) \) is 1 because \( \cos(0) = 1 \).

6. Max and Min Values of \( f(x) = \cos(x) \)
   Max Value of \( f(x) = \cos(x) \) is 1.
   Min Value of \( f(x) = \cos(x) \) is -1
   because \( \cos(x) \) is less or equal then 1 and greater or equal then -1 for all \( x \).

2 Math Homework.

Complete your assignment and submit it to your instructor. Fill in the following table for \( f(x) = \cot(x) \) (each blank in the table is worth 1 point):

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-\pi)</th>
<th>(-\frac{3\pi}{4})</th>
<th>(-\frac{\pi}{2})</th>
<th>(-\frac{\pi}{4})</th>
<th>0</th>
<th>(\frac{\pi}{4})</th>
<th>(\frac{\pi}{2})</th>
<th>(\frac{3\pi}{4})</th>
<th>(\pi)</th>
<th>(\frac{5\pi}{4})</th>
<th>(\frac{3\pi}{2})</th>
<th>(\frac{7\pi}{4})</th>
<th>(2\pi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\cot(x))</td>
<td>(\infty)</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>(-\infty)</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>(-\infty)</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>(-\infty)</td>
</tr>
</tbody>
</table>

Graph the three periods of the cotangent function from the table (graph is worth 10 points):
Fill in the following properties of the graph of the cotangent function. To receive full credit on each question, you must explain your answer (each problem is worth 2 points):

1. Domain of \( f(x) = \cot(x) \).
   Domain of \( f(x) = \cot(x) \): \( \{ x \in \mathbb{R} : \pi n < x < \pi (n + 1) \text{ and } n \in \mathbb{Z} (\text{integer}) \} \), because \( \cot(x) \) is undefined for \( x = \pi n \).

2. Range of \( f(x) = \cot(x) \).
   Range of \( f(x) = \cot(x) \) is all real numbers.

3. Period of \( f(x) = \cot(x) \).
   Period of \( f(x) = \cot(x) \) is \( \pi \), because \( \pi \) is the smallest value \( p \) for which \( \cot(x + p) = \cot(x) \) for all \( x \).

4. The x-intercepts of \( f(x) = \cot(x) \).
   The x-intercepts of \( f(x) = \cot(x) \) are \( x = \frac{\pi}{2} + \pi n \) where \( n \) is an integer (positive or negative), because \( \cot(\frac{\pi}{2} + \pi n) = 0 \).

5. The y-intercept of \( f(x) = \cot(x) \).
   The y-intercept of \( f(x) = \cot(x) \) does not exist.

6. Asymptotes of \( f(x) = \cot(x) \).
   Vertical asymptotes of \( f(x) = \cot(x) \) are \( x = \pi n \) (where \( n \) is integer).
3 Math Homework.

Make sure to show all your work.

\[ f(x) = 2 \sin \left( \frac{2}{3} x \right) \]

1. Graph \( y = 2 \sin \left( \frac{2}{3} x \right) \). Find the amplitude and period of the function.

The amplitude of function \( f(x) = \sin(x) \) equals 1 so in our case amplitude of function \( f(x) = 2\sin(2x/3) \) equals 2.
Period of function $f(x) = \sin(x)$ equals $2\pi$ so in our case period of function $f(x) = 2\sin(2x/3)$ equals $\frac{2\pi}{\frac{2}{3}} = 3\pi$

$$f(x) = -\frac{1}{3}\cos(4x)$$

2. Graph $y = -\frac{1}{3}\cos(4x)$. Find the amplitude and period of the function.
The amplitude of function \( f(x) = \cos(x) \) equals 1 so in our case amplitude of function \( f(x) = -\frac{1}{3}\cos(4x) \) equals \( \frac{1}{3} \).

Period of function \( f(x) = \cos(x) \) equals \( 2\pi \) so in our case period of function \( f(x) = -\frac{1}{3}\cos(4x) \) equals \( \frac{2\pi}{4} = \frac{\pi}{2} \).

\[ f(x) = \tan\left(\frac{1}{2}x\right) \]

3. Graph \( y = \tan(x/2) \). Find the amplitude and period of the function.
The amplitude of function \( f(x) = \tan(x) \) equals \( \infty \) so in our case amplitude of function \( f(x) = \tan\left(\frac{x}{2}\right) \) equals \( \infty \).

Period of function \( f(x) = \tan(x) \) equals \( \pi \) so in our case period of function \( f(x) = \tan\left(\frac{x}{2}\right) \) equals \( \frac{\pi}{\frac{1}{2}} = 2\pi \)

4. Write the equation of the sine function with an amplitude of \( \frac{1}{9} \) and a period of \( \frac{3\pi}{5} \).

As we know the amplitude \( A \) and period \( P \) of the function \( f(x) = \sin(\frac{1}{2}x) \) equals:

\[
A = 1 \times |a| = |a| \\
P = \frac{2\pi}{k}
\]

So in our case \( a = 1/9 \), \( k = \frac{2\pi}{3\pi} = \frac{2}{3} \) and the equation of sine function is

\[ f(x) = \frac{1}{9} \sin\left(\frac{2}{3}x\right) \]

5. Write the equation of the cosine function with an amplitude of 2 and period of \( \frac{3\pi}{5} \).

As we know the amplitude \( A \) and period \( P \) of the function \( f(x) = \cos(kx) \) equals:

\[
A = 1 \times |a| = |a| \\
P = \frac{2\pi}{k}
\]

So in our case \( a = 2 \), \( k = \frac{2\pi}{\frac{3\pi}{5}} = \frac{10}{3} \) and the equation of sine function is

\[ f(x) = 2\cos\left(\frac{10}{3}x\right) \]