Find Fourier standard coefficients:

\[ a_0 = \frac{1}{L} \int_{-L}^{L} f(x) \, dx = \frac{1}{L} \int_{-L}^{0} (-x+1) \, dx + \frac{1}{L} \int_{0}^{L} (-1) \, dx = \frac{1}{L} \left( -\frac{x^2}{2} + x \right) \bigg|_{-L}^{0} + \frac{1}{L} \left( -L \right) = \frac{1}{L} \left( \frac{L^2}{2} + L - L \right) = \frac{L}{2} \]

For positive integer \( n \):

\[ a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos \frac{\pi}{L} nx \, dx = \frac{1}{L} \int_{-L}^{0} (-x+1) \cos \frac{\pi}{L} nx \, dx + \frac{1}{L} \int_{0}^{L} (-1) \cos \frac{\pi}{L} nx \, dx \]
\[ = (-x+1) \frac{1}{\pi n} \sin \frac{\pi}{L} nx \bigg|_{-L}^{0} - \int_{-L}^{0} (-1) \frac{1}{\pi n} \sin \frac{\pi}{L} nx \, dx + \frac{1}{L} \int_{0}^{L} (-1) \cos \frac{\pi}{L} nx \, dx \]
\[ = -\frac{1}{L} \left( \frac{L}{\pi n} \right)^2 (1 - (-1)^n) = -\frac{1}{2} \left( \frac{2}{\pi n} \right)^2 (1 - (-1)^n) \]

\[ b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin \frac{\pi}{L} nx \, dx = \frac{1}{L} \int_{-L}^{0} (-x+1) \sin \frac{\pi}{L} nx \, dx + \frac{1}{L} \int_{0}^{L} (-1) \sin \frac{\pi}{L} nx \, dx \]
\[ = \frac{1}{L} \left( -(-x+1) \frac{L}{\pi n} \cos \frac{\pi}{L} nx \bigg|_{-L}^{0} - \int_{-L}^{0} (1) \frac{L}{\pi n} \cos \frac{\pi}{L} nx \, dx + \int_{0}^{L} (-1) \sin \frac{\pi}{L} nx \, dx \right) \]
\[ = \frac{1}{L} \left( -(-x+1) \frac{L}{\pi n} \cos \frac{\pi}{L} nx \bigg|_{-L}^{0} - \left( \frac{L}{\pi n} \right)^2 \sin \frac{\pi}{L} nx \bigg|_{0}^{L} + \frac{L}{\pi n} \cos \frac{\pi}{L} nx \bigg|_{0}^{L} \right) \]
\[ = \left( \frac{1}{\pi n} \right) ((L+2) (-1)^n - 2) = \left( \frac{2}{\pi n} \right) (2(-1)^n - 1) \]

So the standard Fourier series of function \( c(x) \) is (\( L=2 \))

\[ FS(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{\pi}{L} nx + b_n \sin \frac{\pi}{L} nx \]
\[ = \frac{1}{2} + \sum_{n=1}^{\infty} \left( \frac{2}{\pi n} \right)^2 (1 - (-1)^n) \cos \frac{\pi}{L} nx + \left( \frac{4}{\pi n} \right) (2(-1)^n - 1) \sin \frac{\pi}{L} nx \]

2. Plot several periods of the periodic extension \( \tilde{f}(x) \) of your function (or, respectively, odd or even periodic extension, depending on the series you are working with.)

3. Mark points where the periodic extension is discontinuous. Near them, you will observe Gibbs phenomenon, and at such points, the Fourier series will converge to the average of left and right limits of \( \tilde{f}(x) \).
4. Let $s_N(x)$ be the partial sum of $N$ terms of the Fourier series. We wish to study the convergence of $s_N(x)$ to $f(x)$ at some continuity point $x_0$ (not close to a discontinuity).

Plot several periods of the periodic extension $f'(x)$.
5. In the same manner, study the convergence of \( s_N(x) \) to \( f(x) \) at some continuity point \( x_1 \) close to a discontinuity.

Plot the approximation error at \( x_0 = 1 \), \( \epsilon(N) \) vs. \( N \).
6. In the same manner, study the convergence of $s_N(x)$ to the average of values of $f(x)$ at a discontinuity point $x$.

Plot the approximation error at $x_1 = 0.1$, $\epsilon(N)$.
We find the average of left and right limits of $f^\sim(x)$.

$$\frac{f^+(0) + f^-(0)}{2} = \frac{1 + (-1)}{2} = 0$$

7. Plot the periodic extension $f^\sim(x)$ and several partial sums $s_N(x)$ as functions of $x$. 

![Graph showing periodic extension and partial sums](image-url)