Sample: Astronomy Astrophysics - Astronomy Homework

1. **Solution.**
   
   We have that the gravitational force, which acts on an astronaut, is
   \[
   F = \frac{GMm}{r^2},
   \]
   
   Where \( M \) is mass of Earth, \( m \) is mass of astronaut, \( r \) is radius from center of Earth to astronaut.
   
   On the surface of Earth \( R_E = 6400000 \) m
   \[
   F_E = \frac{GMm}{R_E^2} = gm
   \]
   
   Where \( g = 9.81 \frac{m}{\sec^2} \).
   
   From hence, we have that the \( F = gm \frac{R_E^2}{r^2} \).
   
   Whence, if the space ship of astronaut is in rest relative to Earth in \( R = 51200.00 \) km, his weight is
   \[
   F, \ F = gm \frac{R_E^2}{R^2} = 9.81 \frac{m}{\sec^2} \cdot 91 \text{kg} \cdot \frac{(6400 \text{km})^2}{(51200 \text{km})^2} = 13.9 N.
   \]
   
   If space ship moves by inertia, the weight of astronaut is zero.
   
   In Newton mechanics the mass of astronaut is constant in every point of space and equal 91 kg.

2. **Solution.**
   
   If \( f \) is frequency, \( \lambda \) is wavelength, \( c \) is speed of light, we have that the
\[ f = \frac{c}{\lambda} \]
\[ \lambda = 10000\text{nm} = 10^{-3}\text{cm} \]
\[ c = 3 \cdot 10^{16}\frac{\text{cm}}{\text{sec}} \]

Whence, we have that
\[ f = 3 \cdot 10^{16} \cdot \frac{1}{\text{sec}} = 3 \cdot 10^{13}\text{Hz} . \]

Using the Wien’s displacement law (if object radiates as black body), we have
\[ \lambda_{\text{peak}} T = b = 2897768.5\text{nm} \cdot \text{K} , \]

Where \( \lambda_{\text{peak}} = \lambda = 10\text{nm} \) is the wavelength of peak intensity of radiation, \( T \) is the temperature of object in Kelvin’s.

Whence \( T = \frac{b}{\lambda_{\text{peak}}} = 2898\text{K} \).

We have, that this radiation lies in infrared diapason.

3. **Solution.**

If semi-major axis of orbit of asteroid \( a = 884404000\text{km} \approx 5.9\text{a.u.} \), semi-major axis of orbit of the Earth is \( a_E = 1\text{a.u.} = 149600000\text{km} \), we have according the third Kepler’s law we have that the
\[ \frac{T^2}{a^3} = \text{const} = \frac{(1\text{year})^2}{(1\text{a.u.})^3} \Rightarrow \]
\[ T = \sqrt[3]{a^3} \text{years} = 14.33\text{years} \]

4. **Solution**

If distance between Earth and Mars is \( d = 2.352\text{a.u.} = 351859200\text{km} \)
(\( 1\text{a.u.} = 149600000\text{km} \)). Whence, time, which needs for radio messages to be sent round trip from Mars to Earth to Mars, is \( t = \frac{2d}{c} = 2346\text{s} = 39.09\text{min} \).

Sojourner has to be able to navigate with minimal guidance from Earth because its trip is like step by step. After command Sojourner goes to something position and waits for new command.

5. **Solution**

Distance between Earths and Moon in perigee is \( r_p = a(1 - \epsilon) = 363295\text{km} \) and in aphelion is
\[ r_a = a(1 + \epsilon) = 405503\text{km} , \]

\( a \) is semi-major axis, \( \epsilon \) is eccentricity .

Whence
\[ a = \frac{1}{2} \left( r_s + r_p \right) = 384399 \text{km} \]
\[ \epsilon = \frac{r_s - r_p}{2a} = 0.0549 \]

6. **Solution**

We have that the in non-relativistic limit \( \frac{\Delta \lambda}{\lambda} = \frac{v}{c} \), \( v \) is speed of object.

Here \( \Delta \lambda = 1.234 \text{nm} \), \( \lambda = 10000.000 \text{m} \). If object moves away from observer, \( \Delta \lambda \) and \( v \) have sign "+", object moves towards observer, \( \Delta \lambda \) and \( v \) have sign "+".

Whence \( v = c \frac{\Delta \lambda}{\lambda} = 37.0 \frac{km}{\text{sec}} \). Object moves away from observer.